THE HEATING OF MATERIALS BY A CONCENTRATED ENERGY FLUX UNDER VOLUME ABSORPTION

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An analysis is represented for the process of heating a massive body by concentrated energy flux with volume absorption taken into account. The boundaries of applicability are determined for models of surface and volume absorption. The temperature fields are computed and simple analytical dependences are obtained for the temperature at the center of the heating spot.

The action of a concentrated energy flux (CEF) on a material results in the formation of a heat source with definite space-time characteristics on the surface and within the bulk of a body [1, 2]. This is a surface source for metals in the majority of cases since the degree of laser radiation penetration into a metal is 10^{-6} cm [1]. For a number of other CEF, for instance, for highly-energetic electron beams with 100 kV accelerating voltage and higher, the electrons penetrate into the metal to a depth of tens of μm and more [2]. Moreover, the action of laser radiation on a weakly absorbing medium results in the formation of a heat source decreasing exponentially with the growth of the depth (according to the Bouger law) in the bulk of a body. Consequently, the determination of whether the heat source being formed is surface or volume depends on a number of parameters of the problem including the radius of the heating spot, the action time, etc.

The knowledge of whether the heat source should be considered surface or volume in a specific situation of CEF action on a material is important for a number of technological processes including heat-treatment, welding, etc. [1, 2]. This situation was analyzed first in [3] for a normally distributed heat source, dropping exponentially in the bulk of the material. However, the analysis in [3] was performed only for the cases of small and large times of CEF action while intermediate time cases were not considered.

The purpose of this paper is to determine during what time intervals the surface or volume nature of the absorption of its energy in the material should be considered for a heat source. The deductions obtained can be carried over qualitatively to other energy distribution regularities over the body surface also.

The mathematical formulation of the problem includes a linear heat conduction equation in spatial form without heat losses from the surface and has the form

$$\frac{\partial^2 T'}{\partial x'^2}+\frac{\partial^2 T'}{\partial y'^2}+\frac{\partial^2 T'}{\partial z'^2}=\frac{1}{a}\frac{\partial T'}{\partial t'}-\frac{Aq_0\alpha'}{\lambda\sqrt{k}}\exp\left[-\alpha' z'-k\left(x'^2+y'^2\right)\right],$$

$$0<z'<\infty,\quad \infty<x',\quad y'<\infty,$$

$$\left.\frac{\partial T'}{\partial z'}\right|_0=0,\ T'(x',\ y',\ z',\ 0)=T'(\infty,\ y',\ z',\ t')=T'(x',\ \infty,\ z',\ t')= \tag{1}$$

$$=T'(x',\ y',\ \infty,\ t')=T_0.$$

Introducing the dimensionless variables in the following manner

$$y=\sqrt{k}y',\quad z=\sqrt{k}z',\quad x=\sqrt{k}x',\quad t=kat',\quad \alpha=\frac{\alpha'}{\sqrt{k}}, \tag{2}$$

$$T=\frac{\lambda}{Aq_0}(T'-T_0)$$

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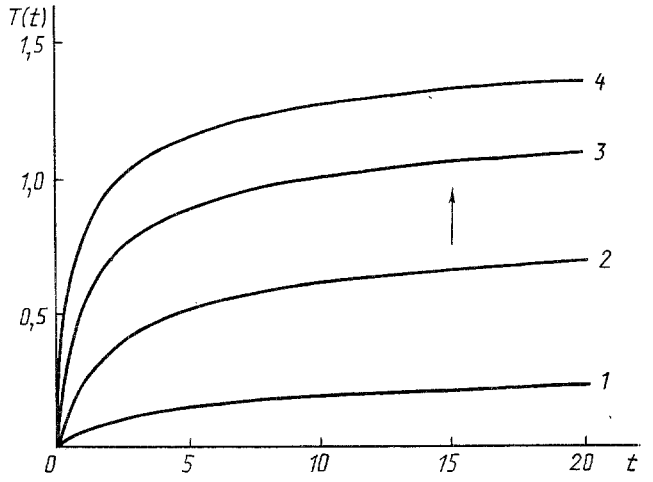

Fig. 1. Temperature at the center of the heating spot: 1) $\alpha = 0.2$; 2) 1; 3) 4; 4) surface absorption.

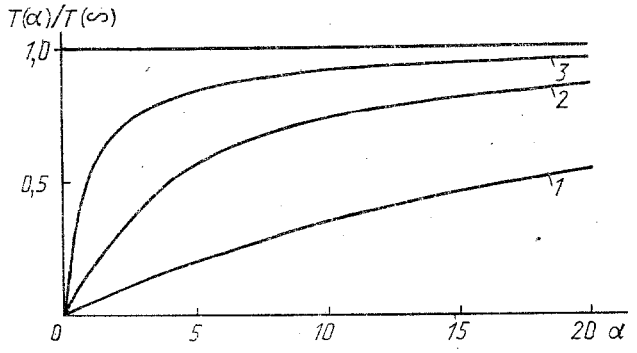

Fig. 2. Dependence of the temperature at the center of the heating spot on the volume absorption coefficient α: 1) at the time $t = 0.01$; 2) 0.2; 3) 20.

and using the symmetry of the problem, i.e., no heat losses from the surface $(\partial T/\partial z|_0 = 0)$, we obtain a solution of the system (1) by using the Green's function

$$T(x,\ y,\ z,\ t) = \frac{\alpha}{16\pi\sqrt{\pi}}\int_0^t (t-\tau)^{-3/2}d\tau \int\int\int_{-\infty}^{\infty} dx'dy'dz' \times$$
$$\times \exp\left\{-\alpha|z'| - (x'^2 + y'^2) - \left[\frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4(t-\tau)}\right]\right\}. \tag{3}$$

The integrals over x', y' and z' are calculated yielding

$$T(x,\ y,\ z,\ t) = \frac{\alpha}{8}\int_0^{2\sqrt{t}} \frac{\xi d\xi}{\xi^2+1} \exp\left(\frac{\alpha^2\xi^2}{4} - \frac{\rho^2}{1+\xi^2}\right) \times$$
$$\times \left\{\mathrm{ch}(-\alpha z) + \frac{1}{2}\left[\exp(-\alpha z)\,\mathrm{erf}\left(\frac{z}{\xi} - \frac{\alpha\xi}{2}\right) - \right.\right. \tag{4}$$
$$\left.\left. - \exp(\alpha z)\,\mathrm{erf}\left(\frac{z}{\xi} + \frac{\alpha\xi}{2}\right)\right]\right\}.$$

where erf(x) is the error integral and $\rho^2 = x^2 + y^2$.

The expression simplifies considerably at the center of the heating spot

$$T(0,\ 0,\ 0,\ t) = \frac{\alpha}{4}\int_0^{2\sqrt{t}} \frac{\xi d\xi}{\xi^2+1} \exp\left(\frac{\alpha^2\xi^2}{4}\right)\mathrm{erfc}\left(\frac{\alpha\xi}{2}\right). \tag{5}$$

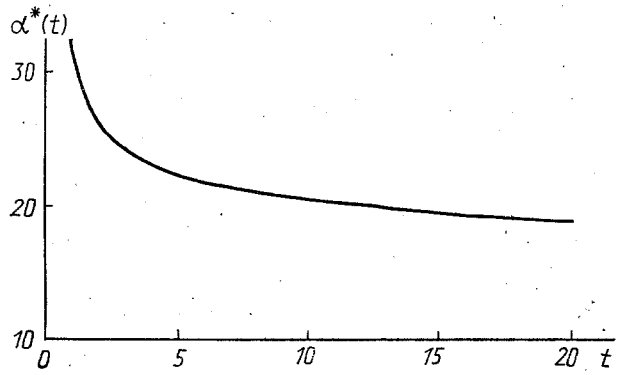

Fig. 3. Time dependence of the volume absorption coefficient α^*.

Analytic expressions are obtained in [3] for the temperature field in the case of large and small times.

The integrals (4) and (5) are analyzed further in the present paper which is a natural continuation of [3].

The time dependences of the temperature for different values of the dimensionless absorption obtained numerically α (curves 1-3) are represented in Fig. 1. For comparison a graph of the temperature field under surface absorption (i.e., ~arctg $2\sqrt{t}$), curve 4, is represented simultaneously. As α grows, the difference between the temperature fields decreases so that starting with $\alpha \sim 20$, the temperature field of volume absorption can be considered equal (to ~5% accuracy) to surface absorption for all heating times, excluding the initial period.

The arrow in Fig. 1 displays the displacement of the temperature field due to a bulk source to the temperature of surface absorption as α grows. Physically, the value α' is constrained by the thickness of the skin-layer that is a quantity ~100 Å for the majority of metals, and by the dimensions of the heating spot.

Figure 2 is analogous to that presented in [3] and reflects the dependence of the surface temperature at the center of the heating spot during volume absorption on the dimensionless absorption coefficient α for different times of action. For convenience, the temperature is normalized to the magnitude of the temperature achievable during surface absorption for the same values of the time. As the heating time increases, the difference between the temperature fields due to volume and surface absorption decreases in a power manner in the form $1/\alpha^2$. It is shown in [3] that such an agreement is observed for large heating times starting with $\alpha \sim 15$. The dynamics of the process of the temperature arriving at the stationary value confirming the deduction [3] is represented in Fig. 2.

The time dependence of the dimensionless absorption coefficient $\alpha^* = \alpha^*(t)$ for which the difference between the temperature fields due to a volume and surface source does not exceed 5% is shown in Fig. 3. If it is impossible to speak about the correlation between the results of surface and volume heating source action ($\alpha^* \gg 1$) for low heating times, i.e., the initial section in Fig. 3, then as the time of action grows the value of the critical quantity $\alpha^*(t)$ is reduced and for $t \geq 20$ practically achieves the stationary value $\alpha^* \approx 15$ [3].

From an analysis of the dependence $\alpha^*(t)$ there also results the fact that for $\alpha^* < 15$ the stationary value of the volume absorption temperature field differs substantially from the surface value (i.e., ~arctg $2\sqrt{t}$). On the other hand, for $\alpha^* > 15$ there always exists such a t^* determined from the condition of a weak change in $\alpha^*(t)$, say (i.e., $|d\alpha^*/dt|_{t = t^*} \leq 0.05$), that the above-mentioned difference is erased for surface and volume absorption.

Therefore, a normal heat source distributed over the bulk of a body according to the Bouger law can be considered surface upon satisfaction of the following conditions: $\alpha > \alpha^*$ and $t > t^*$. For $\alpha < \alpha^*$ a similar deduction is not true and requires taking account of the nature of the bulk absorption.

NOTATION

$T'(x', y', z', t')$ is the temperature field, x', y', z' are space coordinates; t' is the action time; a, λ are thermal diffusivity and thermal conductivity coefficients; A is the absorptivity; q_0 is the radiation flux density at the center of the heating spot; T_0 is the initial temperature; α' is the coefficient of volume absorption in the Bouger law; and k is the coefficient of concentration in the Gauss law.

LITERATURE CITED

1. N. N. Rykalin, A. A. Uglov, I. V. Zuev, and A. N. Kokora, Laser and Electron Beam Treatment of Materials. Handbook [in Russian], Moscow (1985).
2. N. N. Rykalin, I. V. Zuev, and A. A. Uglov, Principles of Electron Beam Treatment of Materials [in Russian], Moscow (1978).
3. N. N. Rykalin and A. A. Uglov, Fiz. Khim. Obrab. Mater., No. 5, 11-14 (1967).

ALGORITHM FOR THE SOLUTION OF THE PROBLEMS OF BODY HEATING DURING RADIATION HEAT TRANSFER

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An algorithm is obtained that permits computation of the optimal metal heating mode in a furnace with minimum fuel consumption during radiation heat transfer.

Metal heating modes assuring minimal fuel consumption during heating while taking account of the predominant influence of the fraction of radiation in the welding zone are examined in [1, 2]. A solution is possible for the problem of selecting the optimal modes on the basis of the method of mainline optimization. To do this the results of [3] are utilized, where a theorem has been presented about the decomposition of the original optimal control problem into three sub-problems: periodic optimization and two auxiliary problems of matching the boundary conditions (problems 1 and 2).

Let the metal heating process be described by the differential equation

$$\frac{dT}{dt} = \frac{T_r^4 - T^4}{\mu} \tag{1}$$

with the boundary conditions

$$T(0) = T_0, \quad T(t_k) = T_k. \tag{2}$$

The functional of process quality characterizing the fuel consumption in the furnace has the form [1]

$$B = \int_0^{t_k} \left(M_0 \frac{T_r^4 - T^4}{T_p - T_r} + M_x \right) dt, \quad (M_0 > 0). \tag{3}$$

The following constraints are imposed

$$T_k < A_2, \quad A_1 \leqslant T_r \leqslant A_2, \tag{4}$$

$$\frac{dT}{dt} \geqslant 0, \tag{5}$$

$$t_k \geqslant t_{\min}, \tag{6}$$

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